Math 201–Midterm (Summer 17)

B. Shayya and H. Yamani

- Please write your **section number** on your booklet.
- Please answer each problem on the **indicated page(s)** of the booklet. Any part of your answer not written on the indicated page(s) will not be graded.
- Unjustified answers will receive little or no credit.

Problem 1 (answer on page 1 of the booklet.)
(8 pts each) Which of the following sequences converge, and which diverge? Find the limit of each convergent sequence.

$$a_n = \left(\frac{n-2}{n}\right)^n \qquad b_n = n\left(e^{5/n} - 1\right) \qquad c_n = \left(1 - \frac{\sin n}{n}\right)^n \left(\frac{(-1)^n}{4} + \frac{1}{2}\right)^n$$

Problem 2 (answer on page 2 of the booklet.)
(8 pts each) Which of the following series converge, and which diverge? When possible, find the sum of the series.

$$\text{(i)} \sum_{n=0}^{\infty} \frac{(-2)^{n+1}}{3^{n+2}} \qquad \text{(ii)} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(\sin n)^9}{n^8} \qquad \text{(iii)} \sum_{n=1}^{\infty} n^{0.1} \ln\left(1 + \frac{1}{n\sqrt{n}}\right)$$

Problem 3 (answer on page 3 of the booklet.)
(15 pts) Find the interval of convergence of the power series $\sum_{n=2}^{\infty} \frac{1}{n \ln n}(x+4)^n$.

Problem 4 (answer on page 4 of the booklet.)
(10 pts) State and prove the n-th term test.

Problem 6 (answer on page 5 of the booklet.)
(11 pts) Find the tangent plane and normal line of the cone $z = -\sqrt{x^2 + y^2}$ at the point $(3, 4, -5)$.

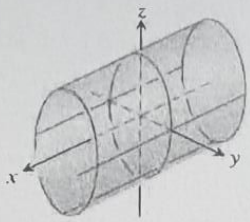
Problem 6 (answer on page 6 of the booklet.)
(i) (8 pts) Use the fact that $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ for all $x \in \mathbb{R}$ to find a power series expansion for the function $f(x) = \cos x$ about the point $x = 0$. Also, find the Taylor polynomial $p_4(x)$ generated by f at the point $x = 0$.
(ii) (8 pts) Use ASET to estimate $\cos(0.1)$ with an error of magnitude less than 10^{-8}. Does your estimate tend to be an over-estimate or an under-estimate?
(iii) (8 pts) Let $p_4(x)$ be the Taylor polynomial from part (i). Use Taylor's theorem to prove that $\cos x < p_4(x)$ for all $0 \le x \le \pi$.

Problem 7 (answer on page 7 of the booklet.)
(12 pts) Match each of the following equations with the surface on the back of this page that it defines.

1.	$x^2 + 2z^2 = 8$	**2.**	$z^2 + x^2 - y^2 = 1$
3.	$9y^2 + z^2 = 16$	**4.**	$9x^2 + 4y^2 + 2z^2 = 36$
5.	$x = z^2 - y^2$	**6.**	$y^2 = x^2 + 4z^2$
7.	$x = -y^2 - z^2$	**8.**	$x = y^2 - z^2$
9.	$x^2 + y^2 + 4z^2 = 10$	**10.**	$z = -4x^2 - y^2$
11.	$z^2 + 4y^2 - 4x^2 = 4$	**12.**	$x^2 = y^2 + z^2$

Your answer should be written in the form **number** $\longrightarrow$ **letter**.

a.



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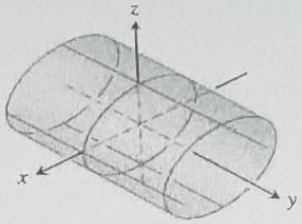

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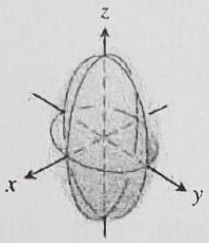

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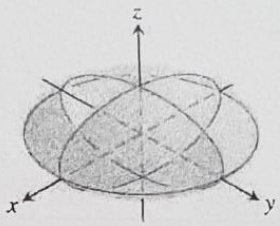

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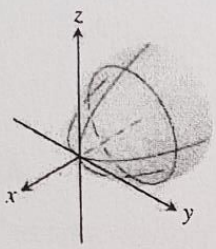

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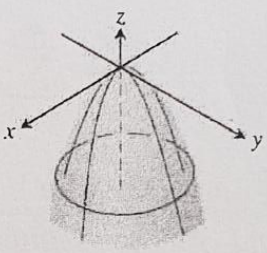

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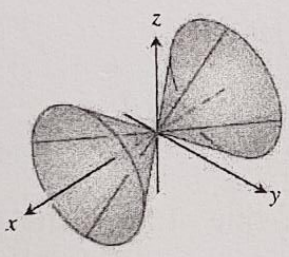

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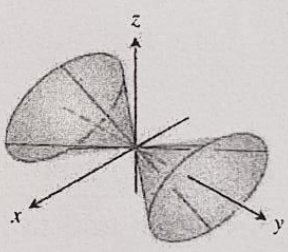
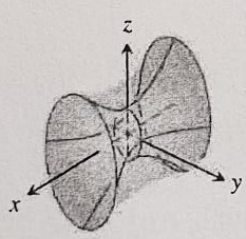


i.



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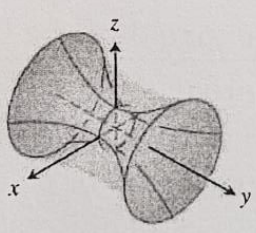

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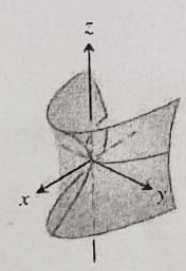

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